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**AGREEMENT**

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## Advanced fatigue testing methods - Part 2: Stiffness method

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## European foreword

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## Introduction

Fatigue failures in materials have been studied for centuries, with early pioneers like August Wöhler noting that repeated loading, even below the static strength of the material, could lead to structural deterioration. Despite decades of research by notable authors, fatigue remains a complex and challenging issue, accounting for most service failures in metallic and composite structures. Therefore, designing structures to withstand cyclic loads without compromising integrity is crucial. However, this process requires conducting numerous fatigue tests to define the appropriate design stress levels for each material and condition. However, determining the fatigue behaviour of metallic alloys and composites through standardised testing methods is often costly and time-consuming. While various techniques have been proposed to expedite testing and enhance the optimisation of materials and components for fatigue resistance, they have not gained wide industry adoption due to limitations in equipment or complex data treatment.

In the present document, an alternative fatigue testing method, named the stiffness method, is proposed to rapidly assess the fatigue resistance of metallic materials with minimal specimens and in a short timeframe. This approach involves monitoring fatigue damage using different variables, such as plastic strain in metallic alloys. These measurements overcome the limitations of other methods by using common extensometers like digital image correlation techniques and contact extensometers. The technique is suitable to readily characterise the fatigue resistance of titanium and aluminium alloys, carbon steels and stainless steels. The estimated endurance limit or fatigue limit and high cycle fatigue curve (S-N curve) obtained through the stiffness method align excellently with values derived from standardised tests. This document describes the experimental procedure and the limitations of the proposed approach.

## 1 Scope

This document describes the procedure for the evaluation of the fatigue resistance of metallic alloys using the stiffness method. This document provides the guidelines for specimen preparation, testing and data post-processing as well as the limitations of the method.

NOTE 1 The test method outlined in this document is designed to rapidly evaluate the fatigue resistance of metallic materials. It is essential to note that the fatigue values obtained through this method are intended for material selection purposes and should not be employed for design considerations.

## 2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes the requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 1099:2017, *Metallic materials — Fatigue testing — Axial force-controlled method*

ISO 12106:2017, *Metallic materials — Fatigue testing — Axial-strain-controlled method*

ISO 12108:2018, *Metallic materials — Fatigue testing — Fatigue crack growth method*

## 3 Symbols and abbreviations

### 3.1 Symbols

b	Fatigue strength exponent
D	Damage variable
dD/dN	Damage progress rate
dΔε/dN	Strain progress rate
ε	Engineering strain
Δε	Total strain range
Δε <sub>e</sub>	Elastic strain range
Δε <sub>p</sub>	Plastic strain range
Δε <sub>p0</sub>	Initial plastic strain range
Δε <sub>pf</sub>	Final plastic strain range
ΔK	Stress intensity range
dσ	Stress increase between each step
k	Slope of the S-N curve
k'	Stiffness of the specimen
K'	Cyclic hardening coefficient
n'	Cyclic hardening exponent
N <sub>f</sub>	Number of cycles to fracture
N <sub>life</sub>	Number of cycles at the knee point
σ	Engineering stress

$\sigma_a$	Stress amplitude
$\sigma_f$	Endurance limit or fatigue limit
$\sigma_{\max}$	Maximum stress
$\sigma_{\text{th}}$	Fatigue damage threshold
$s_x$	Standard deviation
$s_x^2$	Variance
$\sigma_{\text{YS}}$	Yield strength
$\bar{x}$	Mean value

### 3.2 Abbreviations

CDM	Continuum Damage Mechanics
DIC	Digital Image Correlation
NDT	Non-Destructive Techniques
RVE	Representative Volume Element
S-N	Amplitude stress-number of cycles to fracture

## 4 Background

### 4.1 Fatigue phenomenon in metallic alloys

In a metallic material subjected to cyclic loads, the development of fatigue damage can be divided into five distinct stages [1]: small cracks initiation, propagation of microstructurally small cracks, propagation of mechanically/physically small cracks, propagation of long cracks and catastrophic fracture of the material. From a practical point of view, fatigue life is often divided into two main phases: *crack initiation* and *crack propagation*. The crack initiation process includes the nucleation and propagation of small cracks with lengths typically below 1 mm. On the other hand, crack propagation refers to the propagation of long cracks at the macroscopic level which can be detected by non-destructive techniques (NDT).

The fatigue crack initiation phase is described as the transition from the nucleation of a microcrack to the propagation of a macrocrack. This phase cannot be described by fracture mechanics since it requires an existing crack. However, continuum damage mechanics (CDM) is a suitable approach to describe this stage [2,3]. This area of solid mechanics deals with the development of damage in the material, understood as cracks, voids, cavities or any form of material discontinuity at different scales, leading to the deterioration of the mechanical properties of materials. This deterioration of materials is characterised at three different scales: the microscale, the mesoscale, and the macroscale. The nucleation of microcavities or microcracks occurs at the microscale, while the extension of cracks resulting from the coalescence of these microcracks takes place at the macroscale. In between these two processes, the growth and coalescence of microscopic cracks occur at the mesoscale. This cyclic fracture process in materials, along with the deterioration of mechanical properties, is represented by the damage variable (D), which describes the average material degradation within the representative volume element (RVE). The RVE is a small element that CDM uses to divide the material, and its size depends on the material and the mechanical phenomena under consideration. Typically, for metals, the RVE size is on the order of 0.1 mm<sup>3</sup>, while for polymers and composites, it is around 1 mm<sup>3</sup>.

The scalar damage definition ( $D$ ) is bounded between 0 and 1, where  $D = 0$  represents the initial undamaged state, and  $D = 1$  denotes the final fully damaged state. Although this simple CDM approach is sufficient to describe the material deterioration of metals and composites in fatigue [3], some other damage definitions have been proposed by several authors [5]. Such damage can be measured by elastic modulus, electric resistance, ultrasonic wave propagation, temperature, strain response, or microhardness, among others. In CDM, it is essential to precisely know what the final state of the damage process means. When it comes to fatigue, this final state ( $D = 1$ ) corresponds to macroscopic crack initiation, which is the complete fracture of the RVE. In other words, the crack can no longer be considered a small crack and must be described by fracture mechanics concepts.

The distinction between long and small cracks is sometimes taken as 1 mm in length. At this stage, the crack is large enough to no longer depend on the material surface conditions and only depends on the bulk properties of the material. The main issue in fatigue crack propagation mechanics is to establish the equation of crack propagation as a function of a loading parameter. This relationship is given by fracture mechanics ( $\Delta K$ ) and obtained by ISO 12108 for metallic alloys.

The stiffness method employs the strain response to monitor the described fatigue damage evolution in the material during a stepwise increasing stress amplitude test. By tracking this damage, the method enables the assessment of the fatigue properties of the material, such as the S-N curve and endurance limit or fatigue limit, related to CDM and non-propagating microcracks. Remarkably, this is achieved in a short time and with a reduced number of specimens.

## **5 Specimens**

The specimens to be used in the stiffness method shall be in accordance with ISO 1099 requirements.

## **6 Apparatus**

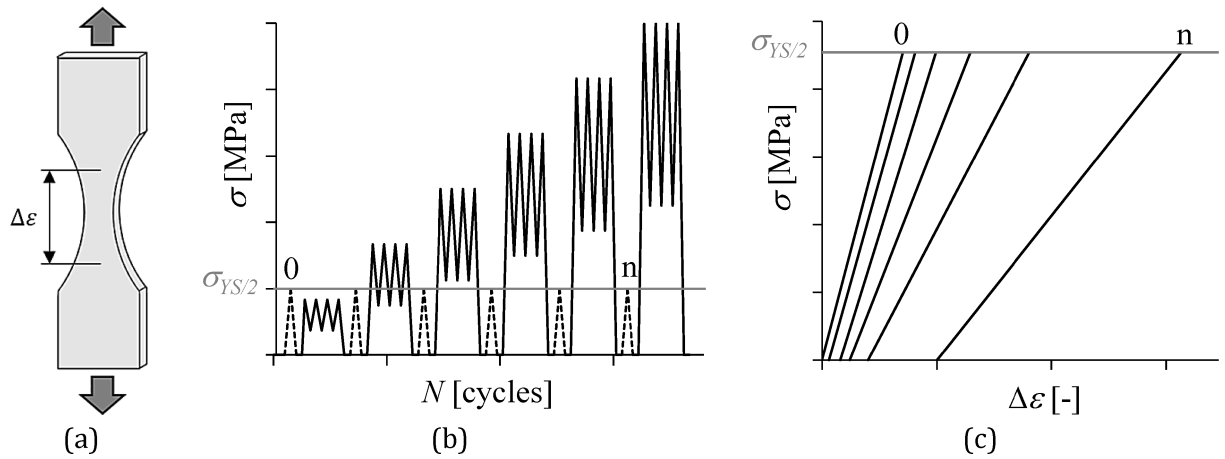
The apparatus and testing machine to be used in the stiffness method shall be in accordance with ISO 1099 and ISO 12106 requirements.

## **7 Procedure**

### **7.1 Endurance limit or fatigue limit assessment**

The methodology employs identical loading conditions of the ISO 1099, encompassing, loading mode, control mode, frequency, waveform, stress ratio, and gripping system. It consisted of the incremental application of fatigue blocks, each spanning 6000 cycles, and an increase of the maximum stress ( $d\sigma$ ) by 12.5 MPa between each successive block. This process is depicted schematically in Figure 1b. A lower number of cycles of each fatigue block or a higher stress increase could lead to inaccurate results due to the induced high plasticity. Conversely, a higher number of cycles of each fatigue block or a lower stress increase could lead to premature fracture of the specimen without gathering sufficient data points for data treatment [6]. The fatigue damage ( $D$ ) is measured through the engineering strain ( $\epsilon$ ) response of the material for a given constant quasi-static engineering stress ( $\sigma$ ). The total strain range ( $\Delta\epsilon$ ) from  $\sigma = 0$  to  $\sigma = \sigma_{YS/2}$  is measured at the outset of the test when the material remains undamaged by fatigue. Subsequently, after each block, it is measured at a quasi-static nominal elastic stress of  $\sigma_{YS/2}$  using a loading-unloading rate of 5 MPa/s. The test is initiated with a  $\sigma_{max}$  far below the expected endurance limit or fatigue limit ( $\sigma_f$ ) such as  $\sigma_{YS/4}$  and concludes when the specimen is completely fractured.





**Figure 1 — Diagram illustrating a) fatigue specimen geometry and strain measurements, b) the stepwise loading procedure, and c) strain taken at quasi-static nominal stress between each fatigue block**

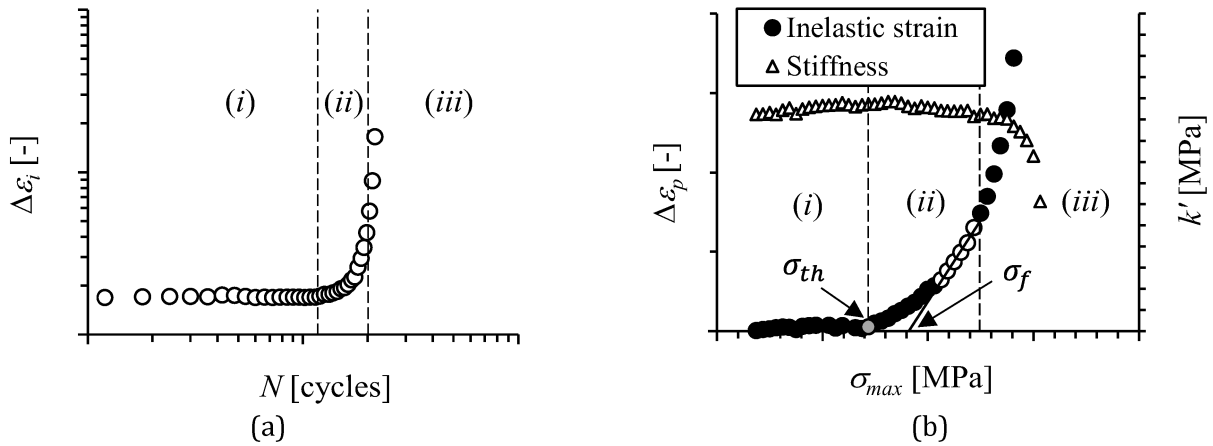
The quasi-static loads sequence, from 0 to  $n$  shown in Figure 1b, allows for the use of a contact extensometer or a 3D digital image correlation (DIC) system to measure the strain. In the case of using a DIC system, the measurement  $\Delta\varepsilon$  must be carried out through a virtual extensometer positioned at the centre of the specimen, encompassing the damaged zone as depicted in Figure 1a.

For monitoring the fatigue damage, only the loading-unloading quasi-static steps between each fatigue block are considered. The  $\Delta\varepsilon$  at  $\sigma = \sigma_{YS/2}$  can be separated into elastic ( $\Delta\varepsilon_e$ ) and plastic ( $\Delta\varepsilon_p$ ) strain measurements. The  $\Delta\varepsilon_e$  is obtained as the average value of the first 5  $\Delta\varepsilon$  values at  $\sigma = \sigma_{YS/2}$  of undamaged material. The accumulated  $\Delta\varepsilon_p$  is the result of the  $\Delta\varepsilon$  at  $\sigma = \sigma_{YS/2}$  minus the calculated  $\Delta\varepsilon_e$  and contains the fatigue damage. The analysis  $\Delta\varepsilon$  allows to identify three regimes associated with fatigue damage as shown in Figure 2a. The first regime (i) is described by an elastic stress-strain behaviour of the undamaged material by small values of  $\Delta\varepsilon_p$ . On the other hand, the third regime (iii) is related to macrocrack propagation and easily identified by a noticeable decrease in the stiffness ( $k'$ ) of the specimen, as depicted in Figure 2b. Therefore, the second regime (ii) includes the small crack initiation and growth shown as an increase of  $\Delta\varepsilon_p$ . Such regimes are described by the strain progress rate, i.e. the increment of  $\Delta\varepsilon$  at  $\sigma = \sigma_{YS/2}$  against the number of cycles (6000 cycles) of each fatigue block ( $d\Delta\varepsilon/dN$ ). The first regime is defined by a  $d\Delta\varepsilon/dN$  value of 0, the second regime is characterised by  $0 \leq d\Delta\varepsilon/dN \leq 10^{-8}$ , and the third regime corresponds to a  $d\Delta\varepsilon/dN$  value greater than  $10^{-8}$ .

The fatigue damage ranging from linear elastic behaviour in regime (i) ( $D = 0$ ) to microcrack coalescence and macrocrack onset at the end of the regime (ii) ( $D = 1$ ) must be quantified according to Equation (1).

$$D = \frac{\Delta\varepsilon_p - \Delta\varepsilon_{p0}}{\Delta\varepsilon_{pf} - \Delta\varepsilon_{p0}} \quad (1)$$

where  $\Delta\varepsilon_{p0}$  and  $\Delta\varepsilon_{pf}$  are the initial and final plastic strain ranges of the regime (i) and (ii), respectively and  $\Delta\varepsilon_p$  is the calculated plastic strain range after each fatigue block. The resulting progression of damage must be employed to identify the data points pertinent to microcrack propagation and the fatigue damage threshold ( $\sigma_{th}$ ) defined by the first  $\Delta\varepsilon_p$  above  $10^{-5}$  related to incipient fatigue damage (Figure 2b). Since the  $\sigma_f$  at the studied regime,  $10^6$ - $10^8$  cycles, is defined by non-propagating microcracks, the points belonging to propagating microcracks defined by a damage rate, i.e. the increment of  $D$  against the number of cycles (6000 cycles) of each fatigue block, ( $dD/dN$ ) of  $10^{-5}$  must be used for linear regression analysis, as illustrated in Figure 2b. The interception of this fitting line with the x-axis determines the stress for non-propagating microcracks, this is the  $\sigma_f$ .

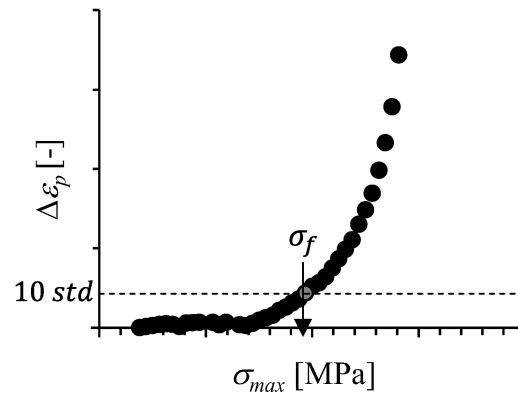


**Figure 2 — a) Plot of total strain range against the number of loading cycles from the stiffness test. Dashed lines separate the three regimes (i), (ii) and (iii) of fatigue damage development based on  $d\Delta\varepsilon/dN$  rates. b) Graph depicting  $\Delta\varepsilon_p$  versus maximum stress to determine  $\sigma_f$ . The solid grey marker highlights  $\sigma_{th}$  and the solid white marker represents the points used for linear fitting to determine  $\sigma_f$ . Additionally, the three regimes are identified, along with the stiffness evolution used to detect macrocrack onset**

The stiffness method procedure can be summarised as follows:

1. Evaluate the  $\Delta\varepsilon$  at  $\sigma = \sigma_{YS/2}$  and calculate the  $\Delta\varepsilon_e$  as an average of the first five  $\Delta\varepsilon$  points (Figure 1b). Calculate the  $\Delta\varepsilon_p$  as the value of  $\Delta\varepsilon$  at  $\sigma = \sigma_{YS/2}$  minus the calculated  $\Delta\varepsilon_e$ .
2. Calculate the  $d\Delta\varepsilon/dN$  using the increase of  $\Delta\varepsilon$  at  $\sigma = \sigma_{YS/2}$  divided by the number of cycles of each fatigue block, i.e.  $dN = 6000$  cycles. Use the calculated  $d\Delta\varepsilon/dN$  to identify the three regimes described in Figure 2a.
3. Identify regime (i) and (ii) and evaluate the accumulated damage  $D$  from Eq. (1) using the calculated  $\Delta\varepsilon_p$ . The  $\Delta\varepsilon_{pf}$  where  $D = 1$  is the last value of regime (ii) identified by a drop in the elastic response  $k'$  (Figure 2b) of the specimen and  $d\Delta\varepsilon/dN \leq 10^{-8}$ .
4. Calculate  $dD/dN$  as the increment of  $D$  between each fatigue block divided by the applied number of cycles in each fatigue block, i.e.  $dN = 6000$  cycles.
5. Identify the values with a  $dD/dN$  in the range of  $10^{-5}$  and use such values to perform a linear regression on the corresponding values of  $\Delta\varepsilon_p$  vs  $\sigma_{max}$ . The intercept of the regression line with the x-axis, i.e. zero plastic strain, is defined as the fatigue limit ( $\sigma_f$ ) (Figure 2b).
6. Additionally, the fatigue damage threshold ( $\sigma_{th}$ ) can be identified by the first  $\Delta\varepsilon_p$  value above  $10^{-5}$ .

A more straightforward approach can be used to determine  $\sigma_{th}$  as the stress level of the first fatigue block that overcomes the threshold defined as 6 times the standard deviation (std) of the values belonging to range (i). Similarly,  $\sigma_f$  for bulk materials can be assessed by the stress level of the first fatigue block that overcomes the threshold defined as 10 times the standard deviation of the values belonging to range (i) as depicted in Figure 3. If the range (i) is not identified, the std is then calculated by using the first 5 points of the calculated  $\Delta\varepsilon_p$ .



**Figure 3 — Fatigue limit determination using the threshold approach for bulk materials. The threshold is calculated as 10 times the standard deviation (std) of the values belonging to the regime (i)**

## 7.2 S-N curve assessment

The strain measurements depicted in Figure 2b belonging to regime (ii) and selected for the linear fitting to determine the  $\sigma_f$ , might be also employed to assess the S-N curve of the investigated material. A generalised form of the power law originally proposed by Morrow [7], is used to describe the stress ( $\sigma_a$ ) – strain ( $\Delta\varepsilon_p$ ) curves depicted in Figure 4 as defined by Equation (2).

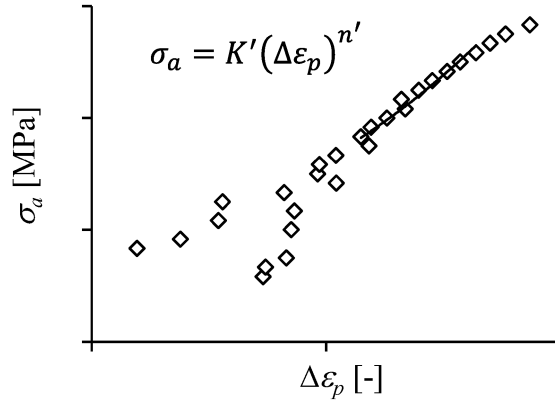
$$\sigma_a = K' (\Delta\varepsilon_p)^{n'} \quad (2)$$

where  $K'$  stands for the cyclic hardening coefficient, and  $n'$  is the cyclic hardening exponent. The S-N curve may be constructed by plotting the number of cycles ( $N_f$ ) for each stress amplitude ( $\sigma_a$ ) following the Basquin model [8] and using the predicted  $\sigma_f$ , the number of cycles at which the fatigue strength is located ( $N_{life}$  at  $10^6$  for high-strength steels or titanium alloys and  $10^8$  for aluminium alloys), and the slope of the S-N curve, represented by  $k$  according to Equation (3).

$$N_f = N_{life} \left( \frac{\sigma_a}{\sigma_f} \right)^{-k} \quad (3)$$

The parameter  $k$  is the inverse of the fatigue strength exponent ( $b$ ), i.e.  $k = 1/b$ . According to Morrow,  $b$  can be calculated from  $n'$  using Equation (4).

$$b = \frac{-n'}{5n' + 1} \quad (4)$$



**Figure 4 — Cyclic stress-strain curve obtained in the stiffness method tests used to define the parameters of the power law. The points used for the linear regression are the same used to define the  $\sigma_f$**

## 8 Analysis of results

Even in the experiments performed and repeated under controlled laboratory conditions, the results scatter. This deviation can be linked to small deviations in the material properties, surface condition, or defects distribution among others. Thus, each result is understood as a random variable with scattering. Due to the reduced number of specimens tested in the stiffness method, usually three, the results are expressed as the mean value ( $\bar{x}$ ) calculated from the test results ( $x_i$ ) and the number of tests  $n$ :

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \tag{5}$$

The mean value is reported together with the standard deviation ( $s_x$ ), which is the square root of the variance ( $s_x^2$ ):

$$s_x^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1} \tag{6}$$

$$s_x = \sqrt{s_x^2} \tag{7}$$

Some statistical recommendations from the staircase procedure are adopted. In this case, it is required that if the ratio of  $s_x$  to the stress increment ( $d\sigma$ ) used in the stiffness test is higher than 1, additional specimens should be tested. This requirement allows for the verification that all tested specimens are governed by the same fatigue mechanism, such as surface crack nucleation, surface defects, or inner defects. However, it is important to note that the primary objective of the stiffness method is to provide a rapid estimation of fatigue resistance. It is not intended for use in design purposes which requires a large number of specimens to consider deviations within the material.

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